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# REVIEW OF RADAR DETECTORS WITH CONSTANT FALSE ALARM RATE

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**ABSTRACT**

Radar detection in environments with high spatial and temporal variability is an issue of great importance. Many efforts are devoted to the development of adaptive techniques that allow robust operation for all possible scenarios. Among the most widely used techniques in this context are those that change the detection threshold adaptively in order to maintain a constant false alarm rate (CFAR). New CFAR detectors appear frequently, so it is useful the permanent update of the state-of-art in this topic. Therefore, this article presents the theoretical framework required by CFAR techniques, as well as a literature review in this matter. Furthermore, the application of non-coherent integration to CFAR detection is included as a distinctive element. Emphasis is made on three different scenarios: homogeneous clutter, multiple interfering targets and transitions in the clutter level. This review will be useful for researchers requiring a brief view of CFAR detection.

**INDEX TERMS:** Radar, CFAR detection, adaptive techniques, non-coherent integration, clutter.

# REVISIÓN DE LOS DETECTORES DE RADAR CON RAZÓN DE FALSA ALARMA CONSTANTE

**RESUMEN**

La detección por radar en ambientes con elevada variabilidad espacial y temporal es una cuestión de gran importancia. Numerosos esfuerzos se dedican al desarrollo de técnicas adaptativas que permitan operar robustamente en todos los escenarios posibles. Entre las técnicas más utilizadas en este contexto se encuentran aquellas que varían el umbral de detección de modo adaptativo con el objetivo de mantener una razón de falsa alarma constante (CFAR). Nuevos detectores CFAR surgen frecuentemente, por lo cual resulta útil actualizar permanentemente su estado del arte. Por tal motivo, este artículo presenta el marco teórico requerido por las técnicas CFAR, así como una revisión de la literatura dedicada a esta temática. Además, se incluye como elemento distintivo la aplicación de la integración no coherente a la detección CFAR. Se realiza énfasis en tres escenarios diferentes: clutter homogéneo, múltiples blancos interferentes y transiciones en el nivel de clutter. Esta revisión será útil para aquellos investigadores que requieran una breve panorámica de la detección CFAR.

**PALABRAS CLAVES:** Radar, detección CFAR, técnicas adaptativas, integración no coherente, clutter.

## 1. INTRODUCTION

Through the analysis of signals reflected by the surveillance region of the radar, commonly called eco-signals, it is possible to detect and classify objects. However, the eco-signals are always contaminated by various types of interference, such as internal receiver noise, unwanted reflections on objects that are not of interest to the radar system and external signals caused by the operation of other electronic devices. Unwanted reflections are the most complex from a practical point of view, since their dynamics are very similar to those of useful eco-signals [1, 2]. For example, it is a challenge to differentiate between the echoes coming from the sea surface, called marine clutter, and those reflected by objects with a very low radar cross-section, such as oil spills or small packages.

In addition to the interference power, its spatial and temporal variability also origins harmful effects on detection, which can be reduced by studying its behavior and preparing the detector to operate properly under the possible scenarios. One of the techniques widely used in practice in order to adapt to interference is the constant false alarm

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rate (CFAR) processing [3]. These techniques were conceived to maintain a false alarm probability ($P_{FA}$) close to the design conditions, regardless of changes in interference power. Note that if other types of changes occur, e.g. in the statistical model, different processing methods should be used [4, 5].

Although there are numerous CFAR alternatives [6, 7], they all share two basic principles: (i) estimate the interference characteristics from a sliding window whose central element is the resolution cell under test and (ii) vary the detection threshold as a function of the above estimate to ensure the desired false alarm probability. The cost of maintaining the $P_{FA}$ is that the probability of detection ($P_D$) will not be maximized, as suggested by the optimal method based on the Neyman-Pearson criterion [3, 8]. For this reason, CFAR techniques are considered suboptimal and will always have a certain loss associated with them. However, their relative ease of implementation and robust behavior have increased their spreading in almost all radar domains [6, 7].

The study and dissemination of the most recent CFAR variants, as well as their grouping according to possible application scenarios, is a permanent necessity. Even when revisions on this subject are available [6, 7], new techniques emerge year after year, so it is useful to keep the state-of-art updated. For these reasons, the objectives of this article are to present the theoretical framework required by CFAR techniques, as well as their updated state-of-art. Also, the application of non-coherent integration to CFAR techniques is included as a distinctive element. This last issue, despite its importance, has only been addressed separately by each of the original works and no single article is found that encompasses the three main approaches.

The following section sets out the foundation for optimal radar detection as a basis for the development of CFAR techniques. The third section addresses CFAR detectors grouped according to the operating scenarios for which they were designed: homogeneous clutter, multiple interfering targets and clutter level transitions. Finally, the fourth section presents the application of non-coherent integration to CFAR techniques.

## 2. OPTIMUM RADAR DETECTION

Detection is one of the fundamental functions of any radar system. Its purpose is to decide whether a certain measurement was caused by the reflection of a target, or was simply an effect of interference. Decisions can be made at different stages of the system, from radio, intermediate and video frequencies up to Doppler processing [9-11]. In the simplest case, samples of the video signal from each range scan are tested individually to decide whether a target is present in the corresponding resolution cell.

Both the interference and the target echoes are described by statistical models. Consequently, the detection process is a typical hypothesis testing problem [12]. For each tested measurement, one of the following assumptions may be made: (i) the measurement is due to interference and (ii) the measurement is the combined result of interference and target echo. The first is called null hypothesis $H_0$ while the second is the alternative hypothesis $H_1$.

The detection logic examines each measurement and decides the hypothesis that "best describes" its origin. Two fundamental questions are implicit in the above quotes: the one describing the involved variables and that allowing the decision to be taken optimally. The first is associated with obtaining the probability density functions (PDFs) of both cases and so much of the detection problem lies in finding suitable models for these. Considering that detection is made from $N$ samples of the data and grouping them in the vector $\mathbf{x}=\left[x_0 \dots x_{N-1}\right]$, the joint PDFs would be denoted as

$$\begin{aligned} &f_{\mathbf{x}}\left(\mathbf{x}/H_0\right) \rightarrow \text{PDF of } \mathbf{x} \text{ given only interference} \\ &f_{\mathbf{x}}\left(\mathbf{x}/H_1\right) \rightarrow \text{PDF of } \mathbf{x} \text{ given interference plus target} \end{aligned} \tag{1}$$

The second issue mentioned above is the selection of the rule that defines unambiguously what is understood by "best" or optimal, in terms of decision making between the two hypotheses. This subject concerns statistical theory and its applications to almost all branches of science are well known [8, 12]. The most used mathematical tool of this theory is the Bayes decision rule, common to all scenarios where decisions are required with a certain degree of uncertainty. Several criteria are obtained from this rule and the Minimum Average Risk criterion [8, 13], which objective is to minimize the average risk or cost per decision. Significant examples of derived criteria are the minimum probability of error, widely used in finance and economics [14, 15]; the maximum likelihood, applied to communications systems [12, 13]; and the Neyman-Pearson criterion, used in radar field. The fundamental motivation for using the latter in radar is the lack of knowledge of the costs associated with possible decisions [8, 12] and the necessity to maintain a constant false alarm rate.

TELEMÁTICA
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Through the Neyman-Pearson's criteria, the rule is designed to maximize the probability of detection ($P_D$) under the restriction of a fixed false alarm probability ($P_{FA}$). The $P_{FA}$ is highly dependent on the specific application and the implications of a "false positive", which can range from tracking a trajectory for a non-existent target, to firing a weapon by mistake. Specifically, the criteria can be approached as follows [8]: to maximize $P_D$ by taking $P_{FA} = \beta$, decide that $\mathbf{x}$ contains a target if

$$\Lambda(\mathbf{x}) = \frac{f_{\mathbf{x}}(\mathbf{x}/H_1)}{f_{\mathbf{x}}(\mathbf{x}/H_0)} > \Lambda_0 \tag{2}$$

where the threshold $\Lambda_0$ must guarantee

$$P_{FA} = \int_{\{\mathbf{x}:\Lambda(\mathbf{x})>\Lambda_0\}} f_{\mathbf{x}}(\mathbf{x}/H_0)\,d\mathbf{x} = \beta \tag{3}$$

The function $\Lambda(\mathbf{x})$ is known as the likelihood ratio and it is a central element to all decision criteria [8, 12-14]. The integration region $\{\mathbf{x}: \Lambda(\mathbf{x}) > \Lambda_0\}$ is the set of $\mathbf{x}$ where the condition $\Lambda(\mathbf{x}) > \Lambda_0$ is met. However, it is rarely necessary to calculate this region explicitly, since the threshold can be obtained from any monotonic function of $\Lambda(\mathbf{x})$. This function allows an equivalent rule to be applied without affecting the result and is referred to as sufficient decision statistics [3, 8]. Denoting this function as $Z(\mathbf{x})$, or simply $Z$, it follows that: a threshold $\Lambda_0$ for $\Lambda(\mathbf{x})$ will correspond to a threshold $Z_0$ for $Z$, thus $Z > Z_0$ implies $\Lambda(\mathbf{x}) > \Lambda_0$. When $Z$ exists, this type of transformation may result in functions whose computation is simpler. The relationship $Z(\mathbf{x})$ presents an essential property: the data appear in the likelihood ratio only through $Z$. Sufficient statistics can be interpreted as a coordinate transformation chosen to put all useful information into a single dimension [13]. Taking the above into account, the optimal decision rule following the Neyman-Pearson criterion can be expressed as

$$\begin{aligned} Z \geq Z_0 &\Rightarrow \mathbf{x} \in H_1 \\ Z < Z_0 &\Rightarrow \mathbf{x} \in H_0 \end{aligned} \tag{4}$$

The specific threshold value that guarantees $P_{FA} = \beta$ could be calculated by (3). However, this equation is not very useful since its evaluation requires multiple integrations in $N$-dimensional space, as well as the explicit definition of the integration region. Since $\Lambda(\mathbf{x})$ and $Z$ are functions of the random vector $\mathbf{x}$, they will also be random variables with their own PDFs. Therefore, the most used alternative approach is to express the $P_{FA}$ in terms of $Z$ and solve the expression

$$P_{FA} = \int_{Z_0}^{\infty} f_Z(Z/H_0)\,dZ = \beta \tag{5}$$

by computing the threshold $Z_0$ that satisfies it [16]. Because false alarms occur when the target is absent, the detection threshold will depend only on the PDF of $Z$ for the case where observations are produced by interference.

As a summary, Fig. 1 shows the essential elements for optimal radar detection. Once the observable variables or input data $\mathbf{x}$ have been defined, models are established for their PDF under the two hypotheses, either by experimentation or by theoretical considerations. Then, the likelihood ratio is considered to define the operations to be performed on the data. From $\Lambda(\mathbf{x})$, and whenever possible, a monotonic function is identified that results in sufficient decision statistics $Z$, which simplifies the calculations. Having a suitable model for the PDF of $Z$ under $H_0$, the threshold $Z_0$ that guarantees the design $P_{FA}$ is determined numerically [16] or analytically [3, 12] through equation (5). The threshold is used by the detector, whose basic function is to apply the rule of (4) to decide the hypothesis that best describes the origin of the observed data.

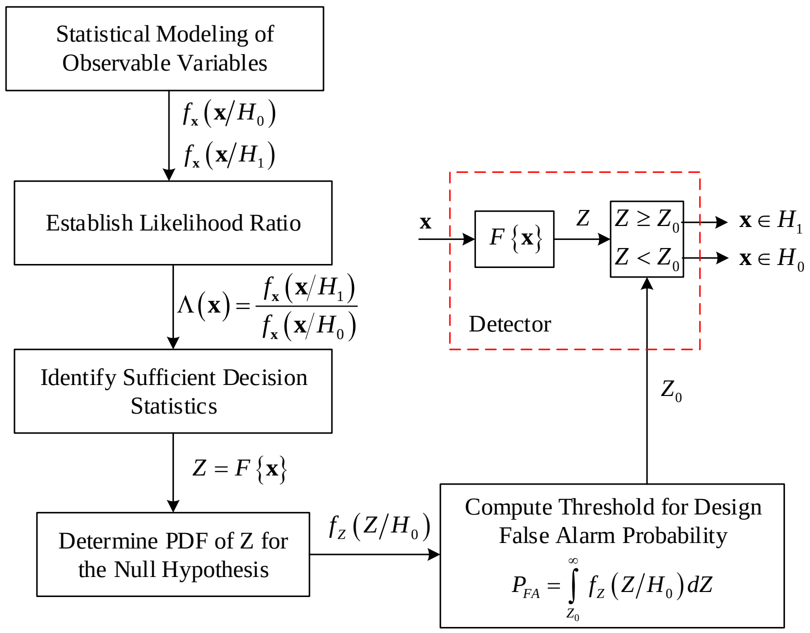


Figure 1: Essential elements for optimum radar detection.

**Optimum detection for a non-fluctuating target in the presence of white noise**

An example of optimum radar detection is the case of a non-fluctuating target through non-coherent integration of $N$ pulses, in presence of additive white Gaussian noise with zero mean value, variance $\sigma^2$ and quadratic demodulator. Grouping the pulse samples in the vector $\mathbf{x} = [x_0 \dots x_{N-1}]$, it is shown [3] that the sufficient decision statistic is given by

$$Z = \sum_{n=0}^{N-1} |x_n|^2 \tag{6}$$

Thus, the squares of the magnitudes corresponding to the $N$ pulses are added up (or integrated according to the radar operator's argot) and the result is compared following the rule (4) with the threshold

$$Z_0 = -\frac{\sigma^2}{A^2} T \tag{7}$$

where $A$ is the non-fluctuating echo amplitude and $T$ must satisfy

$$P_{FA} = \int_{T/\sigma^2}^{\infty} \frac{(Z/\sigma^2)^{N-1}}{(N-1)!\sigma^2} e^{-Z/\sigma^2} dZ = 1 - I\left(\frac{T}{\sigma^2\sqrt{N}}, N-1\right) \tag{8}$$

where $I(\delta, \omega)$ is Pearson's form of the incomplete gamma function given by

$$I(\delta, \omega) = \int_0^{\delta\sqrt{\omega+1}} \frac{e^{-y} y^{\omega}}{\omega!} dy \tag{9}$$

For the simple case of single-pulse detection with $N = 1$, (8) is reduced to

$$P_{FA} = e^{-T/\sigma^2} \Rightarrow T = -\sigma^2 \ln P_{FA} \tag{10}$$

According to (7), optimum detection assumes that $\sigma$ and $A$ are known. From (10) it is shown that $T$ is proportional to the variance or power of the interference, so the $Z_0$ threshold requires knowledge of $\sigma^2$. However, in most cases, the spatial and temporal variability of the environment causes this parameter to be a priori unknown to the radar system.

TELEMÁTICA
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Telemática magazine Vol. 19 No. 3, September-December, 2020, p.78- 90 ISSN 1729-3804

Thus, setting an inappropriate value for $\sigma^2$ induces variations in $P_{FA}$ that become significant. For example, deviations of 2 dB of the real power from the assumed power result in increases of about 3 orders of magnitude in the $P_{FA}$ [3].

Besides, the amplitude of the echoes is virtually unpredictable beyond well-controlled laboratory conditions. Although changes in aspect ratio and target fluctuations could be modeled to some extent [17, 18], it would be extremely difficult to know them in advance and to select the appropriate model for all different scenarios.

## 3. RADAR DETECTION WITH CONSTANT FALSE ALARM RATE

To solve the above problems, detection techniques were developed that calculate the threshold in an adaptive way. Although these are considered suboptimal, since they do not maximize the $P_D$ (because of the lack of prior knowledge about the characteristics of the target to be detected), they do maintain a $P_{FA}$ close to the desired one. This is why they are called Constant False Alarm Rate (CFAR) processing techniques. The general objective that distinguishes them is to estimate the characteristics (power, for example) of the interference by means of target-free "reference channels", so that the threshold is properly adjusted and the $P_{FA}$ is kept at the appropriate levels.

Figure 2 shows a generic CFAR processor for a range profile, although the analysis could be easily extended to several angular sectors and/or several Doppler shift cells. The samples at the output of the demodulator are stored in a shift register that functions as a sliding window [19], which is divided into $U$ leading and lagging reference cells, guard cells (GC) and the cell under test (CUT). Hereafter the term CUT is used interchangeably to refer to the corresponding value of the video signal and to the cell itself, and its meaning becomes clear on the context.

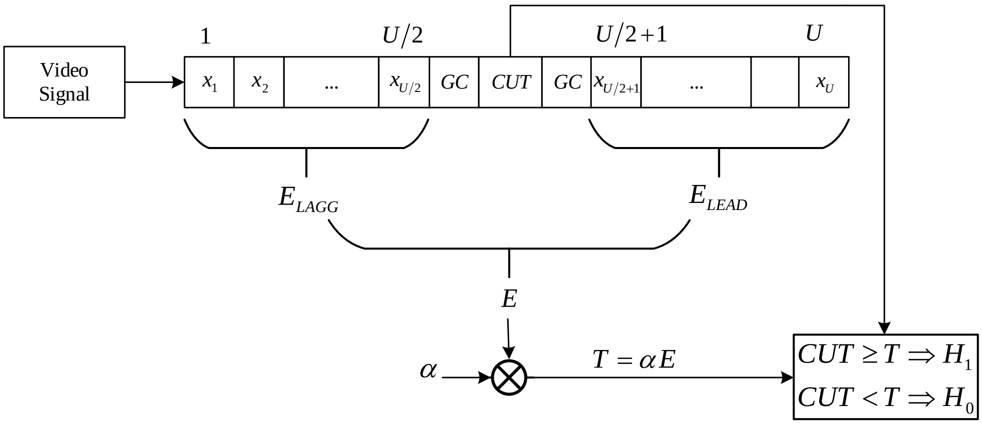


Figure 2: Diagram of a generic CFAR detector.

Generally, the CUT is located in the center of the window and it is the cell compared with the adjustable threshold ($T$) to decide the presence or absence of a target. The reference $U$ cells are used to estimate the interference characteristics ($E$) while the guard cells do not affect this estimate as they may contain echoes associated with a target covering several resolution cells. The window slides so that all cells in the surveillance region take the place of the CUT.

Interference estimates for the leading ($E_{LEAD}$) and lagging ($E_{LAGG}$) windows are generally computed independently. Both are combined to form the total interference estimate $E$, through technique-dependent operations, which may include average, minimum, maximum, etc. The constant $\alpha$ is selected to ensure a certain $P_{FA}$, and is a function of both the $P_{FA}$ and other specific parameters. The threshold $T$ is defined as the product between $\alpha$ and the variable interference estimate $E$, where the points of contact with (10) should be noted. Once $T$ is determined, it is compared with the CUT sample and the decision is made. Then the sliding window is moved by one cell and the process is repeated for all the cells of interest.

The large number of CFAR detectors [6, 7] and the operations they perform are due to the different scenarios presented in real conditions. The simplest of all is the one where the clutter (the most significant type of interference for most applications) can be considered homogeneous. In this case, the statistical characteristics of all reference cells are the same, or at least quite similar, and therefore can be used to provide an adequate estimate of the clutter level and detection threshold. However, in practice there are many situations where heterogeneity prevails. For example, it is common for the reference window to contain targets, which would inevitably affect the clutter estimate with a consequent decline in performance. Another typical situation is transitioning between regions with different clutter, such as storm fronts or coastlines, so the estimate against which the CUT is compared would also be incorrect. For these reasons, the scenarios of homogeneous clutter, multiple targets and clutter transitions, as well as the solutions offered for each case, will be addressed below.

**Scenario with homogeneous clutter**

The scenario with homogeneous clutter can be considered when the samples of the resolution cells are independent and identically distributed. This assumes that there are no interfering targets or clutter level transitions in the window that affect the threshold estimate. For this context emerged the CFAR technique known as CA-CFAR (Cell Averaging CFAR) [20]. The algorithm is relatively simple since it calculates the threshold from the average clutter power estimate according to

$$E_{CA} = \frac{1}{U}\sum_{u=1}^{U}|x_u|^2 \tag{11}$$

Following the logic of Fig. 2, the threshold will have the form $T_{CA} = \alpha_{CA}E_{CA}$, where it is shown that $\alpha_{CA}$ satisfies [3]

$$P_{FA} = \left[1 + \frac{\alpha_{CA}}{U}\right]^{-U} \tag{12}$$

The above equation denotes that the $P_{FA}$ will be independent of the characteristics of the interference ($E_{CA}$). Thus, the CA-CFAR detector maintains the $P_{FA}$ constant, without a priori knowledge of the clutter level. With changes in the clutter, the $P_{FA}$ will remain constant because the threshold $T_{CA}$ is adjusted in the appropriate proportion, depending on the variable estimate $E_{CA}$ and the $\alpha_{CA}$ multiplier. It is precisely this property that defines a detector with a constant false alarm rate [3]. In general, if the $P_{FA}$ expression for a given technique is independent of the characteristics of the interference, as is the case for (12), it is said to satisfy the CFAR condition. The value of $\alpha_{CA}$ is calculated solving (12) for the design $P_{FA}$, given the number of reference cells in the sliding window.

It should be clarified that equations (11) and (12) are valid only for the quadratic demodulator. In general, obtaining the relationship between the CFAR multiplier and the $P_{FA}$ for other types of demodulators is a complex task when considering all variants of CFAR detectors. The emphasis in most of the literature is on the quadratic demodulator [3, 21] and the following analyses will only consider this case.

The performance of the CA-CFAR processor approaches the optimum detector as the number of samples used in the estimate of the interference increases [22-24]. This implies that the number of cells in the reference window $U$ must be increased, which in turn increases the possibility of including heterogeneities that would affect the estimate. Although CA-CFAR remains as the detector of choice in homogeneous conditions, the above contradiction imposes a practical limit on it use. Numerous variants have emerged that improve the performance of CA-CFAR for heterogeneous scenarios like those discussed below.

**Scenario with multiple interfering targets**

The presence of one or more targets in the reference cells, which will be referred to as interfering targets, may cause a target to be masked in the CUT. This phenomenon is divided into two categories: self-masking or mutual masking. The first type is associated with extended targets, those that due to their size take up more than one resolution cell. If an extended target is found in the CUT, the rest of the samples associated with it will correspond to the reference cells and, therefore, will affect the clutter estimate. In this case, the threshold tends to be higher and self-masking may occur. This problem is generally solved by adding guard cells on both sides of the CUT, which depends on the maximum expected target extension and the resolution of the radar. The case of mutual masking occurs when there are interfering targets in the reference window, not associated with the target of the CUT.

Masking is a phenomenon of random nature and will depend among other factors on the window size, the desired $P_{FA}$, the number of interfering targets, and the ratio between the powers of the target at the CUT and the interferer. Several detectors have been designed to solve the above problems. Among these is the FN-CFAR [25] (First-order-difference with Non-coherent integration CFAR) which is designed to eliminate interfering targets by the ordered difference of the samples in reference cells after non-coherent integration. Its performance with homogeneous clutter is quite similar

Telemática magazine Vol. 19 No. 3, September-December, 2020, p.78- 90 ISSN 1729-3804

to that of CA-CFAR and although they claim that it does not require a priori knowledge of the number of interfering targets, indeed the computation of the CFAR multiplier does.

In [26] they propose the SOD-CFAR detector (Second-Order statistics Difference hypothesis CFAR) based on hypothesis testing and second-order differences. In the first step, abrupt changes in reference cells produced by interfering targets are identified and the corresponding samples are discarded. To achieve this, the samples are sorted in ascending order and the index of the sample with the least second-order difference is selected. The remaining samples are then analyzed for exponential distribution using the Shapiro-Wilk test [27] and the reference cells are selected, allowing the clutter to be estimated with homogeneous samples. Although it does not require a priori information about the scenario, its validity is limited to the assumption about the exponential model, so it will fail in case the model is not adequate.

Another variant that shows similar performance to CA-CFAR for homogeneous background and operates robustly in the presence of multiple targets is the two-level detector of [28]. The first level pre-processes the samples by replacing those larger than a threshold with their mean value and then, at the second level, they are subjected to a typical CA-CFAR detector. The fundamental advantage is a reduction in computational cost compared to detectors that require sample order, as is the case with most successful solutions in the multi-target scenario [25, 26, 29, 30].

In [31] they present a detector based on the Grubbs criterion for outliers detection (samples with anomalous behavior), called CAG-CFAR (Cell Averaging Grubbs Criterion CFAR). Its main limitation is that the Grubbs criterion is valid only for Gaussian samples and therefore requires working directly with the samples of the in-phase and quadrature channels. In this way, its usage will be restricted to coherent systems and, in addition, its performance must deteriorate for non-Gaussian interference.

All of the above techniques have recently emerged as alternatives to the most widely used and widespread for the multiple-target scenario. The first to be applied was the CMLD-CFAR (Censored Mean-Level Detector CFAR) [29] and SO-CFAR (Smallest-Of-CA-CFAR) [32]. CMLD-CFAR sorts the samples in ascending order and discards the higher *k*, so they will not contribute to the estimate of the mean clutter power. It is clear that this detector works well as long as the number of interfering targets does not exceed the *k*-value, which should be selected in advance. The SO-CFAR separately compute the leading and lagging window averages and selects the lower as the clutter estimate. In this way, it will be possible to avoid mutual masking, provided that the interfering targets are only in one of the two reference windows.

Without doubt, the most widely used detector for the interfering target scenario is the OS-CFAR (Order Statistics CFAR) [30]. Therefore, it is the reference standard [22, 31] for any new technique. The OS-CFAR sorts the samples in the reference window in an ascending order $\left|x_{(1)}\right|^2 < \left|x_{(2)}\right|^2, \ldots, < \left|x_{(U)}\right|^2$ and selects the *k*-th as representative of the clutter estimate according to

$$E_{OS} = \left|x_{(k)}\right|^2 \tag{13}$$

so that it is capable of suppressing up to $U$ - $k$ interfering targets. In this case, the relationship between the multiplier $\alpha_{OS}$ and the probability of false alarm is given by

$$P_{FA} = kC_k^U \frac{(k-1)!(\alpha_{OS} + U - k)!}{(\alpha_{OS} + U)!} \tag{14}$$

where $C_k^U = U!/k!(U-k)!$ it is the binomial coefficient and the equation must be solved iteratively. As before, the threshold will have the form $T_{OS} = \alpha_{OS} E_{OS}$ and (14) manifests the CFAR property of this detector.

A generalization of the previous sorting and censoring (elimination of samples) detectors is the TM-CFAR (Trimmed Mean CFAR) [33]. Its operation consists of sorting the samples and discarding the largest $k_2$ and the smallest $k_1$ for clutter estimation. This technique, at a slightly higher complexity cost, allows $k_1$ and $k_2$ to be selected in a way that avoids mutual masking and the negative effects of clutter transitions, a topic discussed below. In [34], modifications to the OS-CFAR detector are proposed to work with Pareto distribution clutters, a recently adopted model [1].

**Scenario with clutter transition**

Abrupt changes in ground reflectivity cause the so-called clutter transitions, which increase false alarms and the possibility of masking targets of interest. The occurrence of clutter transitions in the reference window has two fundamental effects on the performance of CFAR detectors. The first is the reduction in detection probability for targets located in the area with the lowest clutter power. The second is associated with increased false alarms in cells near the transition.

The possibility that a transition masks nearby targets depends on the number of cells in the reference window with the highest clutter power and the difference between the two levels [35]. False alarms may increase or decrease depending on the location of the CUT. If the CUT is located in the zone with the lowest reflectivity, false alarms are reduced due to the higher threshold estimate, while the opposite happens when it is in the zone with the highest reflectivity.

Among the detectors designed to operate at clutter transitions is the so-called SKMR-CFAR (Skewness and Mean Ratio CFAR) [36]. Its main characteristic is to decide the clutter estimate from the skewness coefficient and the ratio between the average values of both reference windows. Although they only analyze their performance for heterogeneous clutters with Weibull distribution, the reasoning could be extended to other scenarios. Also for Weibull clutter, the work of [37] combines the non-coherent integration and a double threshold logic (binary integration), with an algorithm that allows to dynamically detect the cell where the clutter transition occurs. Hypothesis tests are performed for the assumed distribution and their computational complexity is relatively high.

Following a different point of view, the paper [38] proposes the Clustering-CFAR detector. This detector uses the CA-CFAR technique, but employs clustering methods to determine the set of cells that, due to their characteristics of homogeneity and proximity to the CUT, could be used to the clutter estimate. The method offers good performance with clutter transitions, although the proposed algorithms require a high computational load. Like Clustering-CFAR, the authors of [24] use a CA-CFAR detector that estimates the clutter from previously selected reference cells, using algorithms based on geographical knowledge of the terrain and other variables. They call the detector KB-CFAR (Knowledge-Based CFAR) and its implementation also requires a high computational cost.

Most of the detectors with a good performance against clutter transitions are of the composite type, briefly discussed in the next section. Most of them [23, 39-43] incorporate as the detector of choice for this scenario the GO-CFAR (Greatest-Of-CA-CFAR) technique [44], which is taken as a reference in this scenario. The GO-CFAR achieves a decrease in false alarms by estimating the clutter power separately for both windows according to

$$E_{GO} = \max\left(E_{LAGG}, E_{LEAD}\right) = \max\left(\sum_{u=1}^{U/2}\left|x_u\right|^2, \sum_{u=U/2+1}^{U}\left|x_u\right|^2\right) \tag{15}$$

where the reason for its name remains clear. The multiplier $\alpha_{GO}$ that guarantees the $P_{FA}$ is determined by

$$P_{FA} = 2\left\{\left[1+\alpha_{GO}\right]^{-U/2} - \left[2+\alpha_{GO}\right]^{-U/2}\sum_{k=0}^{U/2-1} C_k^{U/2-1+k}\left[2+\alpha_{GO}\right]^{-k}\right\} \tag{16}$$

Like with the OS-CFAR, the above equation must be solved iteratively to establish the threshold $T_{GO} = \alpha_{GO}E_{GO}$. Again, compliance with the CFAR condition is verified in (16). The GO-CFAR loss to CA-CFAR with the same window size is estimated to be about 0.3 dB [44] and its hardware implementation using FPGA technology, with some modifications, can be found in [45].

**Other CFAR detectors**

Although the emphasis of the previous sections was on the three detectors taken as reference standards by the scientific community (CA-CFAR, OS-CFAR and GO-CFAR), some numerous techniques and algorithms that attempt to improve their performance exist, especially in heterogeneous environments. Several cases [46-51] focus on providing algorithms that avoid the a priori knowledge of the number of interfering targets and allow automatic identification of the cell where the clutter transition occurs. These single structure detectors are useful because they can operate in different scenarios using the same algorithm. The authors of [52, 53] successfully apply a quite versatile structure that allows their use in several clutter scenarios.

TELEMÁTICA
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Another trend is found in the group of composite CFAR detectors, the first exponent of which was the VI-CFAR (Variability Index CFAR), proposed in [39] and tested for marine environments [54]. The strategy common to this type of detector is based on identifying the most appropriate scenario and selecting a certain detection structure. The identification is made by means of a statistical indicator commonly called variability index, which presents different values for homogeneous and heterogeneous environments.

In this sense, the paper [42] evaluates the performance of the EVI-ASD-CFAR (Enhanced Variability Index Automatic Selection and Detection CFAR) detector, designed for clutter with Pareto distribution. The algorithm selects between GM-CFAR (Geometric Mean CFAR), GO-CFAR and TM-CFAR. A modification of the EVI-ASD-CFAR proposed in [55] involves the Pietra index to select among the same previous detectors.

The paper [41] analyzes the performance against Weibull clutter as a result of combining different detectors for each of the reference windows. The detectors used are CA-CFAR, GO-CFAR, SO-CFAR and OS-CFAR. They propose that CA-CFAR is the choice before homogeneous clutter, for interfering targets the best performance is obtained by combining the lowest of the estimates between OS-CFAR and CA-CFAR, while for clutter transitions it is better to select the highest. The authors of [43] propose the SVM-CFAR detector (Support Vector Machine CFAR), which chooses between the ACCA-CFAR [46] and GO-CFAR algorithms, using a support vector machine. The variability index is used as a feature to train the classifier and recognize the appropriate operating scenario.

Slight variations have also been proposed to the composite detectors, especially concerning the used types [23, 48], the selection logic [56] or the possibility of avoiding the ordering of samples [40] for computation of the variability index. Among the detectors of less diffusion and little general use, there are those based on fuzzy normalization logic [57], goodness-of-fit [58, 59] and Bayesian theory [60, 61].

## 4. APPLICATION OF NON-COHERENT INTEGRATION

Most of the CFAR detectors presented so far decide from a single sample of the video signal for each resolution cell. This is equivalent to detecting once per range scan. However, the accumulation of the samples corresponding to *P* distance scans (known in radar argot as integration) before detection provides a processing gain [3, 62], especially in low signal-to-noise environments. If the integration is done coherently, that is, using the phase of the received pulses, the gain can become *P* times that of a single pulse detection [62]. In the case of non-coherent integration, the gain is between $\sqrt{P}$ and *P* [62]. However, because of its simplicity and low cost, the emphasis is often placed on this type of integration because it is of greater interest to commercial radar systems.

When integrating several ranges scans the value of the $\alpha$ multiplier (see Fig. 2) must be modified, since if the accumulation of samples is not taken into account, then the $P_{FA}$ would be affected. Despite the benefits reported by non-coherent integration, relatively few papers [25, 63, 64] are dedicated to the calculation of CFAR multipliers that guarantee a certain $P_{FA}$, mainly due to the mathematical complexity. All the investigations reported in this direction are limited to the use of the quadratic demodulator and the CA-CFAR, OS-CFAR and GO-CFAR detectors.

Starting with the CA-CFAR detector used in homogeneous clutter environments as previously discussed, the relationship between the $\alpha_{CA}$ multiplier and the $P_{FA}$ is given by [65]

$$P_{FA} = \sum_{p=0}^{P-1} C_{P\cdot U-1}^{P\cdot U+p-1} \frac{\alpha_{CA}^{p}}{\left(1+\alpha_{CA}\right)^{P\cdot U+p}} \tag{17}$$

which must be solved iteratively. Following the same notation as before, *P* represents the number of integrated pulses, *U* the number of reference cells in the sliding window and $C_i^j = j!/i!(j-i)!$ is the binomial coefficient.

For the scenario with multiple interfering targets, the choice is the OS-CFAR detector. In this case, the $\alpha_{OS}$ multiplier will guarantee the $P_{FA}$ if it satisfies [66]

$$P_{FA} = kC_k^U \int_0^\infty \left[ \exp(-\alpha_{OS} y) \sum_{p=0}^{P-1} \frac{(\alpha_{OS} y)^p}{p!} \right] \left[ 1 - \sum_{p=0}^{P-1} \frac{y^p \exp(-y)}{p!} \right]^{k-1} \left[ \sum_{p=0}^{P-1} \frac{y^p \exp(-y)}{p!} \right]^{U-k} \frac{y^{P-1} \exp(-y)}{(P-1)!} dy \quad (18)$$

In the same way that (17), the value of $\alpha_{OS}$ is determined iteratively and the integral must be solved numerically. The $k$ variable is the order of the sample chosen as clutter estimate as proposed by the OS-CFAR algorithm.

On the other hand, for clutter transitions the detector taken as a reference should be the GO-CFAR. The dependence between the $P_{FA}$ and the $\alpha_{GO}$ multiplier is given by [67]

$$P_{FA} = \sum_{l=0}^{L-1} \frac{\Gamma(P+l)}{\Gamma(l+1)\Gamma(P)} \left\{ 2\left( \frac{\alpha_{GO}}{\alpha_{GO}+1} \right)^P \left( \frac{1}{\alpha_{GO}+1} \right)^l - \sum_{v=0}^{L-1} \frac{\Gamma(P+l+v)}{\Gamma(v+1)\Gamma(P+l)} \left( \frac{\alpha_{GO}}{\alpha_{GO}+2} \right)^P \left( \frac{1}{\alpha_{GO}+2} \right)^{l+v} \right\} \quad (19)$$

where $L = UP/2$ and $\Gamma(\cdot)$ is the gamma function [68]. Once again, the value of $\alpha_{GO}$ is calculated iteratively.

## 5. CONCLUSIONS

The spatial and temporal variability of the environment makes the behavior in terms of false alarms unacceptable. In order to estimate the characteristics of the clutter and dynamically adjust the threshold to maintain adequate levels of false alarm, the CFAR techniques were developed. The large number of CFAR detectors and the different operations they perform are due to the different scenarios presented in real conditions.

For the scenario with homogeneous clutter, the technique of choice is CA-CFAR. On the other hand, when the clutter is heterogeneous, the most widely used detectors are the OS-CFAR and the GO-CFAR. The first one is used when there are interfering targets in the surveillance region, while the second is used when there are clutter transitions. These three detectors are reference standards for any new CFAR technique. The addition of non-coherent integration to CFAR detectors provides a processing gain, especially in low signal-to-noise environments.

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**CONFLICT OF INTEREST**

The author declares no conflict of interests.

**CONTRIBUTIONS OF THE AUTHORS**

The author developed all aspect of the work, including the literature review and the preparation of all versions of the article.