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TIME-FREQUENCY-BASED METHODS: APPLICATIONS TO SIGNAL ANALYSIS

Jorge Torres Gómez

[1]Departamento de Telecomunicaciones y Telemática, Facultad de Ingeniería en Telecomunicaciones y Electrónica, CUJAE, La Habana, Cuba
jorge.tg@tele.cujae.edu.cu

ABSTRACT

In communication systems it is of major importance the study of signals not only in time but also in the frequency domain. This is needed to characterize central frequency, bandwidth, channel effects or noise. This is useful to set properly transmitters and receivers. This paper is concerned with the time-frequency methods applied to signal analysis in time and frequency domains simultaneously. Two main approaches are defined, the instantaneous frequency (IF) and the Time-Frequency Representations (TFR) of signals. Unlike the Fourier transform, IF and TFR both allow to describe the evolution of the frequency content of a signal. Thus, non-stationary signals are better analyzed by means of these two approaches than by the use of the Fourier transform. IF is a useful and low complexity technique to analyze monocomponent signals, while TFRs described by the Short-Time Fourier Transform (STFT) and the Time-Frequency Distributions (TFDs) are commonly applied to analyze multicomponent signals. This paper is oriented to describe the main TFR and IF techniques, as well as to discuss the main differences regarding the Fourier transform. The theory behind this techniques is illustrated by some simulations.

KEYWORDS: Instantaneous Frequency, Time Frequency Representation, Time Frequency Distributions, Wigner Ville Transform.

RESUMEN

En los sistemas de comunicaciones resulta de marcada importancia el estudio de las señales no solo en el dominio del tiempo sino además en el dominio de la frecuencia. Este se hace necesario al caracterizar frecuencia central, ancho de banda, efectos del canal o ruido. Lo cual muestra utilidad al configurar apropiadamente los parámetros de recepción y transmisión. El presente artículo está relacionado con la aplicación de los métodos tiempo-frecuencia al análisis de las señales en el dominio del tiempo y de la frecuencia simultáneamente. En esta dirección se desarrollan dos vías principales, el análisis de la Frecuencia Instantánea (IF) y la representación Tiempo-Frecuencia (TFR). A diferencia de la Transformada de Fourier, las técnicas IF y TFR permiten describir la evolución de la frecuencia en el tiempo de una señal. En este sentido, las señales no estacionarias son analizadas mejor por medio de estas dos técnicas que con el uso de la transformada de Fourier. La técnica IF es una herramienta de utilidad y poca complejidad en el análisis de señales monocomponentes, mientras que las técnicas TFR son comúnmente aplicadas al análisis de señales multi componentes. Este artículo está orientado a describir las principales técnicas en relación con FI y TFR, así como señalar las principales diferencias en relación con la transformada de Fourier. Diversas simulaciones ilustran la teoría relacionada con estas técnicas.

PALABRAS CLAVES: Frecuencia Instantánea, Representación Tiempo Frecuencia, Distribuciones Tiempo Frecuencia, Transformada de Wigner Ville.

1. INTRODUCTION

Time-Frequency descriptions are applied to describe the evolution of the frequency content of signals in time. For instance, given a linear modulated tone $x(t) = A\cos(2\pi f(t)\cdot t + \varphi)$ (monocomponent signal),

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in which the instantaneous frequency is given by a linear function of time $f(t) = at + b$ (chirp signal), the goal is to describe in the time-frequency plane the evolution of instantaneous frequency given the signal $x(t)$. This plane must exhibit a time-frequency description as depicted in Fig. 1, in which the red color is denoting a linear relation between time and frequency as expected.

The use of the Fourier Transform does not describe the variations of the frequency components of the signal. Since the Fourier Transform integrates over the entire time axis, then local variations of frequency are not represented on the result. Given this limitation, it is needed to extent the signal analysis not only to the frequency domain, but also to a time-frequency plane. In which non-stationary signals may be jointly described in time and frequency.

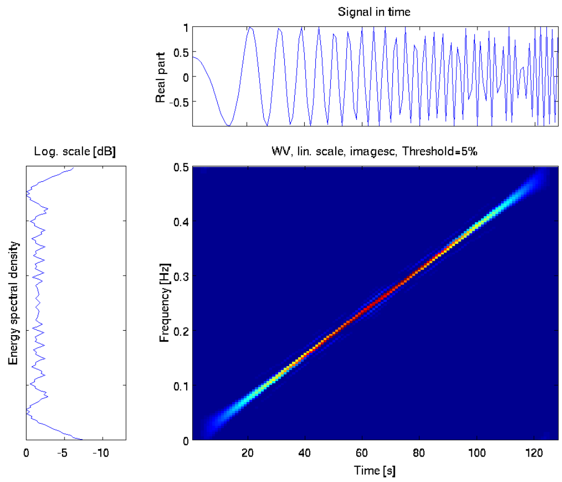

Figure 1: Time frequency description of a chirp signal.

A time-frequency description of signals is comprised by two quantities: the instantaneous frequency (IF) and the spectral spread given by the instantaneous bandwidth. In case of a monocomponent signal the IF is unique, in case of multicomponent signals, a variety of IF values are defined for each tone. A multicomponent signal is defined as a superposition of several tones. To estimate the IF and instantaneous bandwidth, two major approaches are defined: local and global frequency descriptions. By using a global and local frequency description, the time-frequency analysis of the signal attempts to [1]:

1. Track as accurately as possible the spectral variation of the instantaneous frequency.
2. Indicate at each time the measure of the local spectral spread or instantaneous bandwidth.

To perform the above tasks the methods of global frequency description must possess the following properties [1]:

1. Discriminate between stationary and non-stationary signals,
2. Discriminate between monocomponent and multicomponent signals,
3. Break-up a multicomponent signal into its component (also time varying).

Global frequency description can be classified as time-frequency representation (TFR) in two major techniques [2]; Short-Time Fourier Transform (STFT) and Time Frequency Distributions (TFD). These are concepts first addressed by two Nobel Physics Prizes, Gabor and Wigner in addition to the French scientist Ville [3]. The global description makes use of the Fourier Transform. In this case, the waveforms variations define its similarities with the spectral components defined by complex exponentials. The similarities between the given waveform and the complex exponential define the spectrum. Through the graph obtained by the spectrum, it is possible to define the main frequency components of the given

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waveform, but in a global sense, i.e., in a time window. In case of monocomponent signals, the estimated local frequency can be obtained by the first moment or the peak evaluation [2].

On the other hand, the local description is given by the definition of instantaneous frequency. In this case, only one value of the spectral components is defined. Since this is a local description, the variations of waveform do not contribute to several spectral components. The concept of instantaneous frequency determines the main frequency of a monocomponent signal. The second order moment of the instantaneous frequency, which determines the bandwidth, is given by the amplitude variation of the signal [1], which is not a quantity to be obtained by a local frequency description.

The global description takes into account not only the variations of the instantaneous frequency, but also the amplitude variations. On the other hand, the instantaneous frequency describes the variations of main frequency component only. This allows to detect frequency variations, but not detect several tones when a multicomponent signal is analyzed.

The focus on this article is the description of time-frequency methods, which in turn may be used to estimate the frequency content of non-stationary signals. Additionally, these tools may be employed to obtain spectrogram graphs to describe the evolution of frequency components in time when data and noise are mixed together. The rest of the paper is organized as follows: Section 2 summarizes the fundamentals of time-frequency signal description. Section 3 presents the Short-Time Fourier Transform (STFT). Section 4 describes the methods of Time-Frequency Distributions (TFD). Finally, conclusions are presented in Section 5.

2. FUNDAMENTALS OF TIME-FREQUENCY SIGNAL DESCRIPTION

The Time-Frequency description of a given signal aims to describe which signal components are present on a given particular time. To that end two approaches are given, the local and global concepts of ”Frequency”.

2.1. Local Frequency

The local frequency description is described by the instantaneous frequency (IF). This is obtained by representing the signal of interest, denoted by $x(t)$, as an analytic signal given by the rectangular and polar form as [4]:

$$z(t) = x(t) + j\mathcal{H}[x(t)] = a(t)e^{j\phi(t)} \tag{1}$$

where $\mathcal{H}[x(t)]$ represents the Hilbert transform of the signal [1]. The analytic signal is composed by the signal of interest on the real part, $x(t)$, and the Hilbert transform of $x(t)$ in the imaginary part. This representation transforms any real signal $x(t)$ into a complex signal $z(t)$, which offers several advantages provided that operations with exponential functions exhibit to have less complexity than operations with cosine functions. In the frequency domain, by means of the Fourier transform, $Z(f)$ differs from $X(f)$ in that $Z(f) = 0$ when $f < 0$, while $Z(f) = X(f)$ when $f > 0$. That is, $Z(f)$ is composed by the right side of the spectrum of $x(t)$ only.

Based on the polar form in (1), a definition of IF, denoted by $\mathit{\Omega}(t)$, is related to the exponent $\phi(t)$ by [4]:

$$\mathit{\Omega}(t) = \frac{d}{dt}\arg[z(t)] = \frac{d\,\phi(t)}{dt}. \tag{2}$$

The definition of IF in (2) is obtained by the application of the stationary phase principle [5]. The principle states that the Fourier integral:

$$Z(\omega) = \int_{-\infty}^{\infty} z(t)e^{-\mathrm{j}\omega t}dt = \int_{-\infty}^{\infty} z(t)e^{-\mathrm{j}2\pi f t}dt = \int_{-\infty}^{\infty} a(t)e^{\mathrm{j}[\phi(t)-2\pi f t]}dt \tag{3}$$

reaches the maximum value whenever the phase remains stationary, that is $\frac{d}{dt}[\phi(t) - 2\pi f t] = 0$, there is no time dependence. Which in turn leads to the definition of IF in (2). This definition is named to be local in the sense that only the instantaneous values of phase are considered to determine the IF value, nor the amplitude and nor additional time instant of $x(t)$ are analyzed.

2.2. Global Frequency Description

The global description is defined by the Fourier Frequency and brings the Time-Frequency Representation (TFR) of signals. Measurement of frequency components from a global point of view takes into account the similarities between the signal of interest $x(t)$ and the complex exponential e^{-jwt} through the use of the Fourier transform as:

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-jwt}dt \tag{4}$$

These similarities, measured by the quantity $X(w)$ is obtained by the correlation between $x(t)$ and e^{-jwt}. The quantity $X(w)$ brings the frequency content of $x(t)$, as long as $x(t)$ is similar to a pure tone of frequency value w, considering this tone of constant amplitude an defined from $-\infty$ to ∞.

However, the use of the Fourier expression in (4) limits the analysis of frequency components when the frequency varies in time. The variable ω does not reflect instantaneous values of the local frequency in time. Thus, by means of $X(\omega)$ it is not possible to derive the instantaneous frequency $\Omega(t)$, then the analysis of non-stationary signals is limited. In this regard two major TFR techniques have been developed; Short-Time Fourier Transform (STFT) and Time-Frequency Distributions (TFD).

Frequency tracking applications demand that the TFR behaves in a manner that can be related to the standard notion of frequency. In this regard, these TFR techniques, given by the quantity $\rho_z[n, k]$ in the discrete case or $\rho_z(t, \omega)$ in the continuous case, are defined to have the following properties [2]:

- $\mathcal{P}1$ The TFR should be real valued since a complex-valued two-dimensional function is impossible to interpret as a surface.
- $\mathcal{P}2$ Summation in time should yield the signal energy spectral density, that is $\sum_{n=0}^{N-1} \rho_z[n, k] = |X[k]|^2$.
- $\mathcal{P}3$ Summation in frequency should yield the instantaneous power $\sum_{k=0}^{N-1} \rho_z[n, k] = |x[n]|^2$, where $X[k]$ is the DFT of $x[n]$ [6].
- $\mathcal{P}4$ The time support of the signal should be preserved in the TFR:
  If $x[n] = 0$ for $n < n_1$ and $n > n_2$,
  Then $\rho_z[n, k] = 0$ for $n < n_1$ and $n > n_2$.
- $\mathcal{P}5$ The frequency support of the signal should be preserved in the TFR:
  If $X[k] = 0$ for $k < k_1$ and $k > k_2$,
  Then $\rho_z[n, k] = 0$ for $k < k_1$ and $k > k_2$.
- $\mathcal{P}6$ For interpretation as an energy distribution, the TFR must also be non-negative definite:
  $\rho_z[n, k] \geq 0$

where $x[n]$ and $X[k]$ represent the samples of $x(t)$ and $X(\omega)$, respectively [6]. Properties $\mathcal{P}2$ and $\mathcal{P}3$ establish that the projections of the TFR on time and frequency planes must satisfy the same properties of the energy distribution in time and frequency, respectively. For instance, the variances of the time

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and frequency content of the signal, given by the quantities $|s[n]|^2$ and $|X[k]|^2$ remains on the TFR information given by $\rho_z[n,k]$. The quantity $\rho_z[n,k]$ expresses how the time variance varies in frequency and the frequency variance varies in time. The time variance, expressed by $|s[n]|^2$, can be analyzed as a function of frequency. Conversely, the frequency variance, given by $|X[k]|^2$, can be analyzed as a function of time, which in turn is named the instantaneous bandwidth.

The TFR technique can be analytically representable by the generalized Cohen Class [2] (discrete formulation):

$$\begin{aligned} \rho_z[n,k] =& 2\,\mathfrak{F}_{m\to k}\{B[-n,m] *_{(n)} k_z[n,m]\} \\ =& \sum_{m=-M}^{M}\sum_{p=-M}^{M} B[p-n,m]\cdot \\ & \cdot z[p+m]z^*[p-m]e^{-j2\pi\frac{mk}{N}} \end{aligned} \tag{5}$$

where:

- $\mathfrak{F}_{m\to k}$ represents the Fourier transform applied on the time index m to give a discrete spectrum in k.
- $*_{(n)}$ is the convolution operation in the time index n.
- $k_z[n,m] = z[n+m]z^*[n-m]$ is the bilinear product.
- $z[n]$ represents the analytic signal.
- $B[n,m] = N\mathfrak{F}^{-1}_{l\to n}[f(l,m)]$ represents the time-lag kernel function.
- $f(l,m)$ represents the Doppler-lag kernel function which characterizes the individual members of the class.
- N is the signal length.
- $M = \frac{N-1}{2}$

The relation in (5) expresses that any TFR can be obtained by properly smoothing any other TFR. The differences between TFRs is determined by the kernel function $B[n,m]$ [1]. Furthermore, the quadratic relation in (5) is provided in terms of energy units. The time-frequency distribution $\rho_z[n,k]$ designates the energy content of the signal in the time frequency plane [7].

In case that the kernel function $B[n,m]$ is defined to be independent of the signal, then the TFR is said to be bilinear [8]. Unfortunately, to accomplish with Property 6 above implies that some other properties can not be satisfied simultaneously in case of bilinear TFRs. It is not possible to obtain true energy distributions in the time-frequency plane. The representations that satisfies Properties from 1 to 5 are named TFDs, while the STFT satisfies the Property 6, but not the Properties 2 and 3 or the Properties 4 and 5 [2].

Proper TFDs are those defined to accomplish with all the properties $\mathcal{P}1$ to $\mathcal{P}6$. In this case the kernel function, defined by $B[n,m]$, must be dependent of the signal to be analyzed [8, 9, 10, 11].

2.3. Differences and Similarities between the Local and Global Frequency Descriptions

The definitions given in (2) and (4) for the IF and Fourier Transform, respectively, have the following differences [12]:

1. The Fourier frequency ω is an independent variable, while the instantaneous frequency $\Omega(t)$ is a function of time.

2. The Fourier frequency is associated with the Fourier transforms in (4), while the instantaneous frequency is associated with the Hilbert transforms.
3. The Fourier frequency is a global quantity defined with respect to the entire signal, while the instantaneous frequency is a local descriptor of the signal at a particular instant in time.
4. The weighted mean of the Fourier and instantaneous frequencies remains equal, while the variance, along with other higher moments, do not in general coincide. The second order moment satisfies the inequality:

$$\overline{(\omega-\overline{\omega})^2} \geq \overline{(\Omega(t)-\overline{\Omega(t)})^2} \tag{6}$$

   The equality holds only in the case where the signal's envelope is a constant.

Besides, the instantaneous frequency IF and the frequency estimated by the TFR does not coincide, except for the case when $B[n,1]=\delta[n]$. In other cases, the instantaneous frequency will be filtered by the kernel function $B[-n,1]$.

On the other hand, the variance of $X(\omega)$ and $x(t)$ are related by the Heisenberg uncertainty principle as $\varphi_t \cdot \varphi_\omega \geq \frac{1}{2}$, which in turn exhibits the limitations of the frequency analysis of signals. Small values of φ_t cause large values of φ_ω. Then, signals limited in time are not concentrated in frequency. In this case, to determine the instantaneous frequency $\Omega(t)$ is limited by the use of $X(e^{j\omega})$. In case we want to analyze some portions of the signal in time, then the spectrum will be spread in frequency and the uncertainty in determining the instantaneous frequency will be large.

The uncertainty principle applies only to the Fourier transform pairs $x(t)$ and $X(\omega)$, which are functions solely of t and ω, respectively. The relation is not applied to a function of time and frequency as $\rho(t,\omega)$, it is only applied to the marginals in time and frequency given by $|x(t)|^2=\int_{-\infty}^{\infty}\rho(t,w)\mathrm{d}w$ and $|X(e^{jw})|^2=\int_{-\infty}^{\infty}\rho(t,w)\mathrm{d}t$, respectively.

The uncertainty principle does not constraint any other univariate or joint moments of the time-frequency representation of the signal given by $\rho(t,\omega)$. This not implies any restriction on the correlation between time and frequency [8].

3. Short-Time Fourier Transform

Short-Time Fourier transform is defined through the use of the Discrete Fourier Transform (DFT) algorithm applied to consecutive sections of the signal.

This method is applied in two steps:

1. The window of interest, denoted by $w[n]$, is applied to the received signal $y[n]$ in different time instants m as:

$$y_w[n,m]=w[n-m]\cdot y[n] \tag{7}$$

2. Then, the DFT algorithm is computed to the windowed signal $y_w[n,m]$ by:

$$STFT(k,m)=\frac{1}{N}\sum_{n=0}^{N-1}y_w[n,m]e^{-j\left(k\frac{2\pi}{N}\right)n} \tag{8}$$

Thus, by shifting this window using the time index m, it is expected to have the evolution in time of the frequency content of the signal, denoted by k, when the DFT is consecutively computed as shown in Fig. 2. The STFT is also implemented by several windows as the Rectangular, Hanning, Hamming, Bartlett, Kaiser-Bessel [6] among others.

Additionally, the STFT can be summarized by the general formulation in (5) given the following expression of the kernel $B[n,m]$ [2]:

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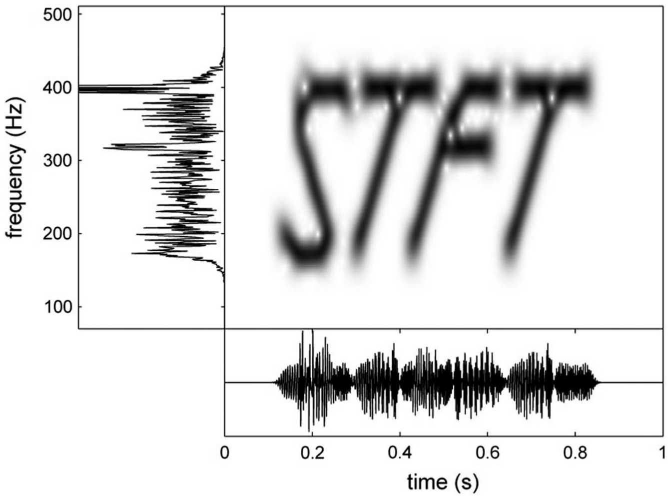

Figure 2: STFT technique [13].

$$B_{STFT}[n,m] = \frac{1}{M} \Pi_{Q-|m|}(n), \tag{9}$$

where Π represents the rectangular pulse, and M the length of the window given by $M = 2Q + 1$.

Additional solutions of STFT techniques are reported to have an improvement in regard to leakage noise or detection in low SNR regimes. In [14] a new method is devised to obtain a better estimation of the active power. In this solution, DFT coefficients are averaged in time to reduce the leakage effect. The coefficients are averaged using the a_l terms of Rife-Vincent window. The report in [15] improve detection in a low SNR environment through the use of smoothed polynomial terms. Additionally, short-term Fourier Bessel expansion can be used to describe a signal in the time-frequency plane as described in [7]. Besides, further studies to improve time-frequency resolution by modifying the window length is also addressed in [16].

4. Time-Frequency Distributions (TFDs)

The time evolution of the frequency content of the signal is described in a similar way to statistical considerations using a bivariate joint probability distribution. Time-Frequency Distributions (TFDs) is a subclass of TFR.

In order to describe a TFD by $\rho_z[n,k]$ in (5), the function $B[n,m]$ must satisfy the following conditions based on the properties $\mathcal{P}1$ to $\mathcal{P}4$ [2]:

- $\mathcal{P}1 \rightarrow B[n,m] = B^*[n,-m]$
- $\mathcal{P}2 \rightarrow \sum_{\forall_n} B[n,m] = 1$
- $\mathcal{P}3 \rightarrow B[n,0] = \delta[n]$
- $\mathcal{P}4 \rightarrow B[n,m] = 0,\ |n| > |m|$

Due to the shape and magnitude satisfied by $B[n,m]$ to accomplish with the above restrictions, these kernels are named the Bowtie functions. A variety of distributions have been defined to satisfy the above properties. as described in Table 1. Additional TFD's are given by the Dolph-Chebyshev S transform in

Table 1: Time Frequency Distributions (TFDs) [1, 10].

TFD	B[n,m]
Wigner-Ville (WVD)	$\delta[n]$, $m \in \left[-\frac{M-1}{2}, \frac{M-1}{2}\right]$
Choi-Williams	$\frac{\sqrt{\frac{\sigma}{\pi}}}{2m} e^{-\sigma \frac{n^2}{4m^2}}$
Zhao-Atlas-Marks (ZAM)	$w[m] \Pi\left(\frac{n}{\frac{2m}{a}}\right)$
Smoothed WVD	$\frac{1}{P}$, $n \in \left[-\frac{P-1}{2}, \frac{P-1}{2}\right]$, 0 otherwise
Rihaczeck-Margenau	$\frac{1}{2}[\delta[n+m] + \delta[n-m]]$
Born-Jordan-Cohen	$\frac{1}{\lvert m\rvert+1}$, $\lvert m\rvert \leq \lvert n\rvert$, 0 otherwise

[16], Compact Support Kernels [17], Smooth-Windowed WVD [18] also applied to power quality analysis and Neural Networks [19], Empirical Mode Decomposition [20] and Compact Support Kernels [21].

The first time-frequency distribution was defined by Wigner in 1932, and then introduced to signal analysis by Ville in 1942. This distribution is the best employed to concentrate energy in the time-frequency plane, but the worst in regard to the cross-term effect. The rest of the distributions attempts to minimize this effect by modifying the kernel function. This kernel function attenuates the cross-terms and passes the true time-frequency components [10].

In case of monocomponent signals of constant amplitude $z[n] = Ae^{j\phi[n]}$, the TFR representation by means of (5) yields [22]:

$$\rho[n,k] = 2\pi A^{2\alpha}\delta(k-\phi[n]) *_k W[k] *_k \mathcal{F}_{m\to k}\left\{e^{jQ[n,m]}\right\}. \tag{10}$$

Relation in (10) implies that a monocomponent signal is represented in the time-frequency plane by a function centered at $\phi[n]$, given by the Delta function. However, this is smoothed by a window function $W[k]$ and a spread factor $Q[n,m]$. The spread factor $Q[n,m]$ is caused by the higher order phase derivatives, this is when $\frac{d^n\phi(t)}{dt^n} \neq 0$ for $n > 1$. The value of α denotes the order of the distribution. For a quadratic distribution $\alpha = 1$, this is the case of the WVD.

The term $Q[n,m]$ is suppressed in case of linear frequency modulated signals (chirps). However in case of non-linear modulated signals higher order distributions can be used to reduce the spread factor. These distributions are derived from the WVD as [22]:

$$\rho[n,k] = \sum_{m=-\infty}^{\infty} w_L[m] z^L\left(n + \frac{m}{2L}\right) z^{*L}\left(n + \frac{m}{2L}\right) e^{-jk\frac{2\pi}{N}m} \tag{11}$$

where $w_L[m]$ is a window function and L is an even integer. In addition, in case of a fast varying IF, a complex-time distribution is also considered by:

$$\rho[n,k] = \sum_{m=-\infty}^{\infty} z(n+\frac{m}{4}) z^{-1}(n+\frac{m}{4}) z^{-j}(n+j\frac{m}{4}) z^{j}(n+\frac{m}{4}). \tag{12}$$

Since these distributions are defined by non-linear operations, a cross term appears in the time-frequency plane. This leads to a miss interpretation of the frequency content of the signal.

Given a signal composed by two mixed tones of frequencies Ω_1 and Ω_2. It is expected to have energy around these two frequency values, named autocomponents. However, when the TFR is obtained by means of TFDs, a cross-term appears at mixed values of Ω_1 and Ω_2, named cross-components. This, when analyzed on the time-frequency plane leads to new frequency components of the signal. A TFD is said to have good performance when high degree of cross-component suppression and autocomponent concentration is obtained [10].

In fact, TFDs are determined to have a concentrated representation of the time-frequency characteristics of the signal. In this regard, some measures are defined to determine this property by [19]:

- Ratio of norms based measures,
- Shannon & Rényi entropy measures,
- Normalized Rényi entropy measure,
- Jubisa measure,
- Time and Frequency resolutions [23].

A good example of TFD formulation without cross-terms is given by the SM transform as [24]:

$$SM(t,\Omega) = \frac{1}{\pi}\int_{-pi}^{\pi} P(\theta) STFT(t,\Omega+\theta) S^{*}(t,\Omega-\theta)\mathrm{d}\theta. \tag{13}$$

where S is a column vector containing the STFT, as given in (8), with a varying M window size. The relation given in (13) relates the STFT and the WVD, taking into account the values of $P(\theta)$. In case that $P(\theta) = 1$, then $SM(t,\Omega)$ represents the WVD. When $P(\theta) = \pi\delta(\theta)$ then SM gives the STFT.

Figure 3 exhibits the behavior of the SM transform when applied to a multicomponent signal. The multicomponent signal is comprised by two damped tones at different IFs and a tone of constant amplitude and constant IF as:

$$x[n] = \sin\left(\frac{\pi}{8}\right) e^{-\frac{(n-n_0)^2}{\sigma^2}} + \sin\left(\frac{\pi}{4}\right) e^{-\frac{(n-n_0)^2}{\sigma^2}} + \sin\left(\frac{3\pi}{4}\right) \tag{14}$$

where the length of the sequence $x[n]$ is given by $N = 2^{10}$, $\sigma = 100$ and $n_0 = \frac{2^{10}}{2}$. This is analyzed using 4 different SNR values. The code is provided in [25] by the authors of reference [24].

The result shown in Figure 3 exhibits the peaks of the 3 different signals. Even in the presence of high AWGN noise, it is possible to detect the presence of the signal and estimate the IF for each component.

5. DISCUSSION AND CONCLUDING REMARKS

The interest in obtaining a time-frequency representation of signals is useful when we want to visualize the evolution of frequency in time. This is commonly used to study non-stationary signals described by frequency components which are time-varying. In this case a joint time-frequency description must be provided to analyze this type of signals.

To obtain a time-frequency representation of a given signal, local or global description of frequency have to be obtained. In the former case the analytic signal definition is employed. In the latter case two major approaches are employed: Short-Time Fourier Transform (STFT) and Time-Frequency Distributions (TFD).

The local frequency description is limited to the analysis of monocomponent signals. In case of multicomponent signals, the peaks detection of several tones is unattainable since the resulting phasor has an

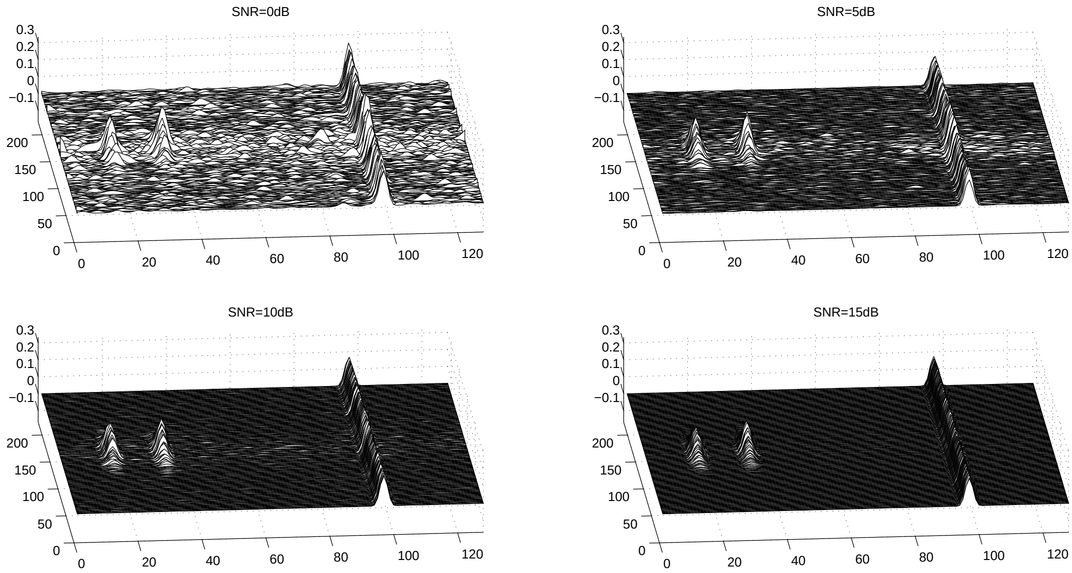

Figure 3: SM Transform of a multicomponent signal.

instantaneous phase with mixed components of several tones. Multicomponent signals must be analyzed by TFDs [1].

The TFR can be used to detect the instantaneous frequency of monocomponent signals. This can be attainable by computing the first moment or peaks of the obtained description. However, no benefit is obtained from this approach when monocomponent signals are analyzed, since a much simpler algorithm is obtained when the derivative of the analytic signal is derived [2].

From the mentioned TFRs, the only two distributions in which the first-order moment is equal to the instantaneous frequency are given by the WVD and the Choi-Williams Distributions (for large values of σ) [1]. However, in both cases the spread about the IF estimated is increased when non-linear frequency modulated signals are considered. To reduce this effect, higher order TFRs are preferred.

The STFT and TFDs suffer from the presence of cross-tems [2]. This effect appears when a multicomponent signal is analyzed. In case that the signal of interest is comprised by two or more sinusoidal components, then the mixture of this terms will appear as unwanted oscillations on the TFR graphs. This leads to a wrong interpretation of the frequency content. Large cross-term are obtained with the use of WVD when a multicomponent signal is considered. This causes major problems in determining the frequencies of interest.

STFT suffers from leakage noise due to the side-lobes of the applied window. The small the window in time is, the greater the effects of the side-lobe will be. Although the use of short windows is convenient, since a local frequency description needs to be obtained, its application is limited by the leakage noise. Thus, the length and the side-lobe amplitude of the window represents a trade-off when a local frequency description is obtained by means of STFT techniques.

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AUTHOR AFFILIATIONS

Jorge Torres Gómez received the Bachelor degree in Telecommunication and Electronic Engineering in 2008, the Master degree in Telecommunication Systems in 2010 and the PhD degree in 2015 from the Technological University of Havana, CUJAE. He is currently working at CUJAE, Telecommunications and Electronic School, Department of Telecommunications and Telematics, where he is an Auxiliar Professor.