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# DEMODULATORS FOR BFSK SIGNALS BASED ON MATCHED FILTERS: A SURVEY.

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## ABSTRACT

The present article addresses several reported techniques for demodulating BFSK signals through the use of correlator receivers. A survey in regard to the different correlators used as matched filters is addressed and the main parameters are discussed. Besides, several BFSK waveform demodulators are described and the results are illustrated by means of simulations. A detailed description of the reported demodulators is given by means of the block scheme and complexity of the proposed methods is also analyzed. Advantages and disadvantages in regard to the reported solutions are also analyzed.

**KEY WORDS:** BFSK, Matched filter, Quadricorrelators.

## RESUMEN

El presente artículo describe las distintas técnicas de demodulación de señales BFSK reportados en la literatura científica que emplean receptores de correlación. Se describen las distintas técnicas de correlación que se emplean como variantes de filtro adaptado así como sus principales parámetros. Además se describen distintos demoduladores de señales BFSK y sus resultados se ilustran mediante simulaciones. Se aborda el principio de funcionamiento de cada técnica y la complejidad de su implementación mediante el análisis de su diagrama en bloques. Se describen además las ventajas y desventajas alrededor de cada tipo de demodulador.

**PALABRAS CLAVES:** BFSK, Filtro adaptado, Correladores cuádricos.

# INTRODUCCIÓN

The digital modulation BFSK (Binary Frequency Shift Keying) is usually employed for several applications in the field of digital communications. This modulation is used for transmitting information over power lines (PLC) [1], [2], mobile communications [2], [3], spread-spectrum systems [4], [5] and for satellite communication [6].

The BFSK waveform is produced by the variation of the instantaneous frequency $w_c$, switching between two values, $w_0$ and $w_1$, depending on the binary information to be transmitted. Figure 1 depicts an example of the BFSK waveform and the information transmitted. The signal in figure 1 comprises two symbols, each one at different frequencies and in the interval of $N_{samp}$ samples, matching the low frequency with the low level in Figure 1 a), the high frequency with the high level of the binary information. Each interval of $N_{samp}$ samples is determined by the analytic expression $A_c\, cos(w_c\, n + \varphi_c)$, which represents the sampled version of $A_c\, cos(2\pi f_c\, t + \varphi_c)$, being $A_c$ the amplitude of the received tone, $w_c = \frac{2\pi f_c}{f_s}$ the instantaneous frequency, $\varphi_c$ the phase of the received signal and $n$ is defined in the interval $[0\ \ N_{samp} - 1]$.

Several options are available for demodulating this type of signals. The schemes reported are mainly based on the energy detector, loop phase systems and matched filters. Examples of these type of systems can be founded in [7], [8], [9]. On the other hand, the solution using ideal matched filters, with the Quadrature Correlator, has a significant advantage since it applies the optimum solution for detecting signals in noise [7]. This is the best option under low SNR, but knowledge about parameters of the incoming signal, such as symbol time (for synchronization), and carrier frequency and phase, is required. That is, the synchronicity with the time symbol, the frequency and the phase. The ideal matched filter is, in theory the optimum solution, unlike its practical implementation.

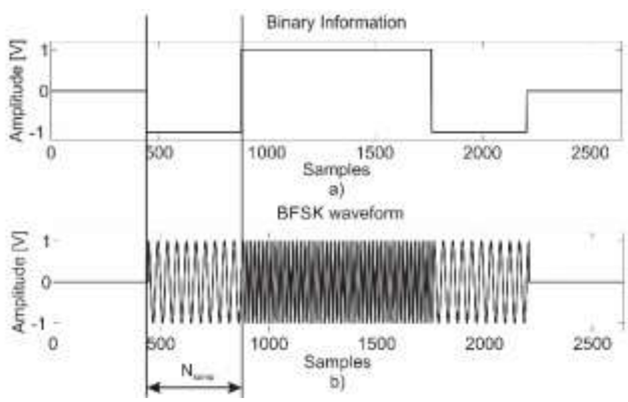


**Figure 1: Graphical example of the BFSK modulation. a) Binary Information. b) BFSK signal. Parameters:$w_0 = 0.3562$ [rad], $w_1 = 0.1425$ [rad], $N_{samp} = 441$ Samples, sampling frequency $f_s = 44100$ Hz.**

Matched Filters are implemented through a correlation between the incoming signal and the local tone generated in the receiver. The ideal matched filter is a synchronous and coherent detector, since the operation of correlation is performed only in the interval of a symbol and the local tone must be a proportional and in-phase copy of the incoming signal. However, there are solutions that although use a correlator procedure, are non-coherent and asynchronous. This paper addresses those solutions, since the use of high complexity blocks for synchronization is avoided and the solution is optimal among the non-coherent and asynchronous schemes.

Several detectors for demodulating BFSK waveforms are reported by using correlators in a noncoherent and asynchronous scheme. Those are the Quadricorrelator [10], the Balanced Quadricorrelator [11] and the Quotient Detector [12]. The detectors comprise a correlator and a detection block as depicted in figure 2. In this respect, the present article refers to the different schemes for implementing the correlator block and demodulators of BFSK signals.

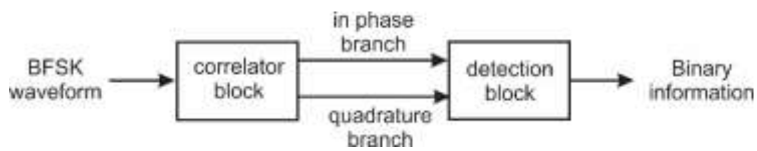


**Figure 2: General scheme of receivers based upon correlation for BFSK signals.**

The rest of the paper is organized as follows: Section 2 summarizes the different schemes in regard to the matched filters solutions. Section 3 presents the reported demodulators through the use of correlators in a noncoherent and asynchronous scheme. Finally, conclusions are presented in Section 4.

**MATCHED FILTER SCHEMES**

The cross-correlation procedure between the input signals, the received and the local waveforms allows a detection with the highest SNR. This operation, also known as matched filter, minimizes the energy of the noise at the receiver output while remains constant the energy of the signal of interest [7].

The detection of signals in AWGN by matched filtering is implemented through a filter with a transference function equal to the conjugate spectrum of the transmitted signal. In case of BFSK signals, this operation is analogous to perform the cross correlation between the input signal and the basis functions of the modulation [7]. This way, the identification of the $f_0$-frequency symbol is equivalent to the detection of the functions $\cos(2\pi f_0 t)$ and $\sin(2\pi f_0 t)$, since the received signal can be decomposed into the linear combination of them, in other words, $A_c\, cos(2\pi f_c\, t + \varphi_c) = A_c \cos(\varphi_c)\cos(2\pi f_0 t) - A_c \sin(\varphi_c) \sin(2\pi f_0 t)$. The detection of the $f_1$-frequency symbol is accomplished in the same way.

The block scheme for implementing the correlation between the basis functions $\cos(2\pi f_0 t)$ and $\sin(2\pi f_0 t)$ and the modulation signal $A_c\, cos(2\pi f_c\, t + \varphi_c)$, is depicted in figure 3, this is known by the Quadrature Correlator [7]. The cross correlation is accomplished using a multiplier and an integrator, that works at the interval of $T_s$ seconds.

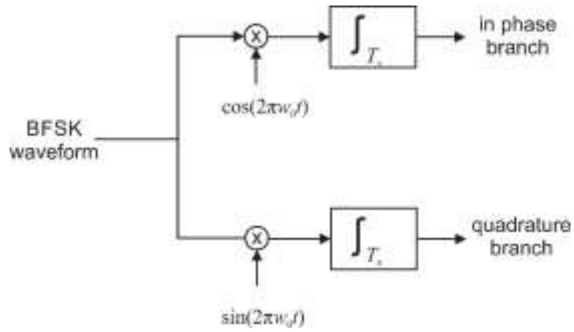


**Figure. 3. Circuital scheme for the Quadrature Correlator.**

Several implementations of correlators are reported. These solutions replace the ideal integrator of Figure 3 by a lowpass filter or by an accumulator. Quantifiers can also be used in order to simplify the correlation procedure. Then, the correlation schemes can be classified into 5 types [13]: Direct correlator, Digital correlator, Stieltjets correlator, Modified digital correlator and Modified Stieltjets correlator. Except for the Direct correlator, the others correlators apply some distortion in the input signal in order to simplify the arithmetic operations. The performance of this correlators is measured by the following issues [14]:

- The ratio between the correlation function at the output of this these versions of correlators and the ideal one.
- SNR at the output, SNRo

**Direct Correlator**

Using the Direct Correlator the input signal is not distorted. The Direct Correlator is rightly called as "correlator" because the post-detection SNR is higher than that of any other correlator. The SNR parameter for the Direct Correlator is presented as:

$$SNR_{Direct\ Correlator} = \frac{\sigma_s^2}{\sigma_n^2 2WT} \tag{1}$$

Where $\sigma_s^2$ and $\sigma_n^2$ represent the energy of the information signal and noise, respectively, and the factor $WT$ represents the time-bandwith product of the information signal. This correlator can be implemented through two schemes: The Analog Correlator and the Discrete Correlator. The Analog Correlator replaces the integration procedure by lowpass filters, as depicted in Figure 4 a). This type of correlator is used by the digital receiver analyzed in Section 2. Besides, the Analog Correlator can be implemented digitally, in particular, the lowpass filters [15]. On the other hand, the Discrete Correlator processes samples of the input signal and replaces the integrator in figure 3 by an accumulator, as depicted in Figure 4 b). In this case, the SNR paramater is equal to (1).

Others correlator are compared with the Direct Correlator by means of Sensibility as follows:

$$Sensibility = \frac{SNR}{SNR_{Direct\ Correlator}} \cdot 100\% \tag{2}$$

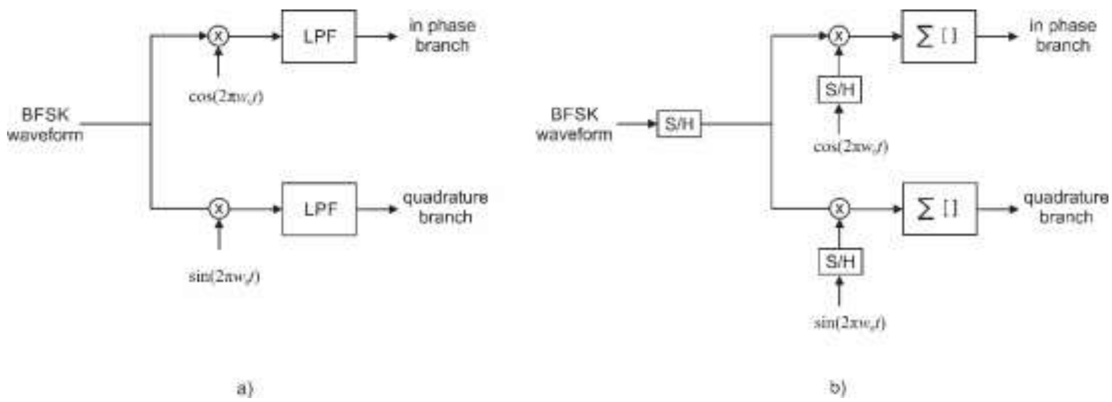


**Figure 4: Circuital scheme for the Quadrature Correlator.**

**Digital Correlator**

The Digital Correlator quantifies the input signals and implements a digital filtering operation instead of the analog one, as shown in Figure 5 a). This quantification process modifies the correlation function between the quantified received signal $y_q$ and the quantified local generated signal $x_q$.

A numerical approximation of the relationship between the correlation functions $R_{xq\,yq}$ and $R_{xy}$, as the correlation between the quantified and non-quantified input signals, respectively, is the following:

$$R_{xq\,yq} \approx R_{xy} + \frac{q}{12} \tag{3}$$

In (3), $q = \frac{1}{2^n}$ and $n$ is the total number of bits for representing the quantity $x$ or $y$. This approximation tends to equality exact when the quantification is performed by employing just 3 bits in the quantification procedure. On the other hand, the Sensibility is a function of the total of levels and the sampling frequency. In case that 7 bits are employed, an efficiency over 94% is obtained.

An special case of quantification is obtained through the Polarity Coincidence Correlator (PCC), which applies a hard clipping to the input signals. That is, just 1 bit precision and only two levels for representing the input signal. Figure 3 b) depicts this scheme where the lowpass filter is replaced by an accumulator with transference function $H(z) = \frac{1}{1-z^{-1}}$.

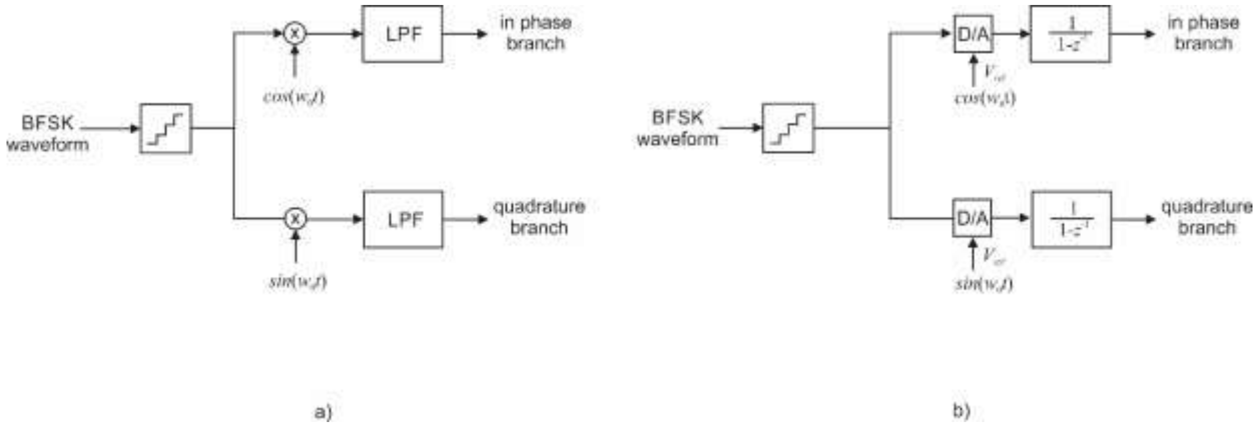


**Figure. 5. Block Scheme of Digital Correlator. a) Multilevel Quantifier. b) One bit Quantifier.**

The relation between the original correlation function (without quantification) and the quantified version, is described by:

$$\rho_{xq\ yq} = sin^{-1}\left(\frac{\pi}{2}\rho_{xy}\right) \tag{4}$$

Where $\rho_{xy}$ and $\rho_{xq\ yq}$ represent the normalized correlation function without quantification and with quantification respectively. With this relation it is possible to recover the original correlations function, and then to cancel the effects of quantification. In this regard, the block $\frac{2}{\pi}sin[\cdot]$ in 3 b) is employed. The procedure employed for quantifying the input (see 3 b)) leads to a reduction of hardware complexity. Indeed, the multiplier can be replaced by a NOR gate and the lowpass filter, by an accumulator. This is one of the most employed correlator due to the ease of implementation. However, this system exhibits a Sensibility not higher than 80%, then, implementation of the sine wave function is required in order to work with the original correlation, increasing the hardware cost of the correlator.

A practical implementation can be attained by limiting the amplitude of symbols in the input signal by means of the function $f(x) = \frac{1}{2\pi}\int_0^x e^{-\frac{1}{2}\frac{t^2}{d^2}}dt$, where $d$ is equal to the symbol time. In this case, the relation between the normalized quantified and the original correlation functions is given by:

$$\rho_{xq\ yq} = \frac{d^2}{2\pi}sin^{-1}\left(\frac{\rho_{xy}}{1+d^2}\right) \tag{5}$$

The integration is performed by lowpass filters. This configurations is suitable to be applied when the bandwidth of the received signal is higher than the sampling frequency of the digital circuit.

## Stieljets Correlator

The Stieljets Correlator quantifies just one of the two input of the correlator. Figure 4 a) depicts the Stieljets Correlator scheme. Besides, the multiplier can be replaced by a Digital to Analog Converter, when the reference signal of the D/A block is fed as indicated in figure A.4 b) as depicted in 4 b). The

Sensibility of this type of correlator is around the 90%, this with 3 bits, this can be improved when the number of bits are increased. The Stieltjets Correlator can also be implemented through the so-called Relay Correlator. In this configuration just one of the input is quantified with two levels, while the second one remains as the original. In this scheme, the sensibility is around the 80% and with higher sampling frequency this is increased to 90%.

**Modified Correlator**

Modified versions of the Digital and Stieljets Correlators can be implemented by adding noise source to the input. The idea is to obtain a gaussian statistics distribution in the input of the correlator. Then, with this procedure it is possible to obtain in the correlator output the result of the ideal correlation function. This solution emulates an ideal matched filter. Besides, in case of the PCC correlator it is avoided the use of the block $sin[\cdot]$ by this way.

**ASYNCHRONOUS RECEIVERS FOR BFSK WAVEFORM BASED UPON CORRELATORS**

The asynchronous receivers reported through the use of correlators have in common the Analog Correlator scheme and have different kind of detection blocks. All of them perform operations for extracting the difference between the frequency of the received signal and that of the local oscillator in the correlator block. The receivers based on this technique are the following:

1. Quadricorrelator
2. Balanced Quadricorrelator
3. Quotient Detector

**Quadricorrelator**

The Quadricorrelator scheme is depicted in figure 6. It comprises a correlator and a detection block given by a discrete-time differentiator and a multiplier. The output of this system is modulated in amplitude by the difference of frequency between the received symbol and the frequency stored, that is $\Delta w = w_c - w_L$.

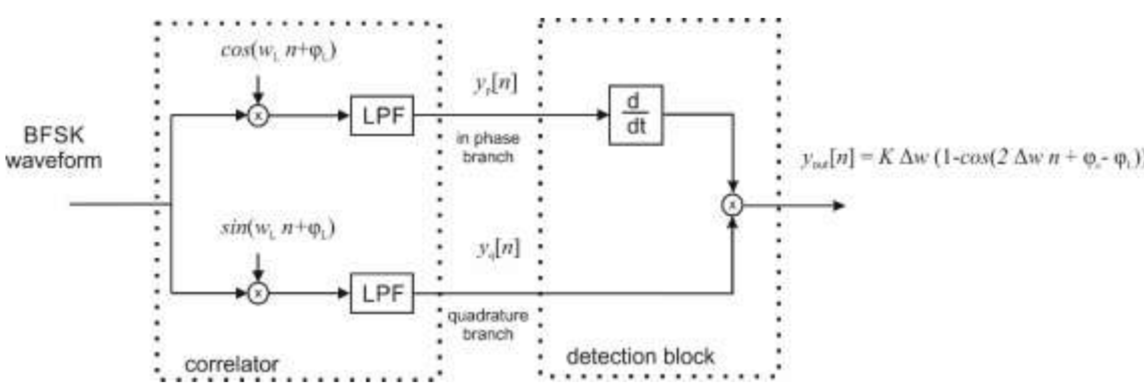


**Figure 6: Quadricorrelator scheme.**

Figure 7 depicts the simulation performed with the Quadricorrelator. It can be seen in 1.4 b) that a filtering process is needed in order to remove the oscillating term. Figure 1.4 c) shows the results obtained when a low pass filter with cutoff frequency $w_c = 2\Delta w$ is applied.

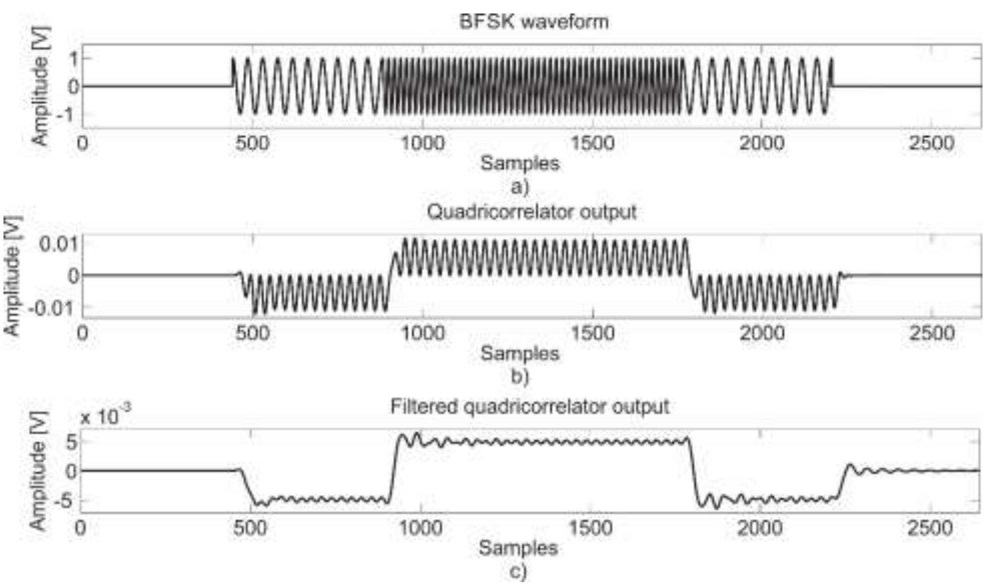


**Figure 7: Graphical example of the Quadricorrelator performance a) BFSK signal. b) Quadricorrelator output. c) Lowpass filtered Quadricorrelator output. Low pass filter implemented through an elliptic IIR filter of 9th order, Differentiator filter implemented by a Kaiser window of fifth order. Parameters: $w_0 = 0,3562$ [rad], $w_1 = 0,1425$ [rad], $N_{samp} = 441$ samples, sampling frequency $f_s = 44100$ Hz.**

**Balanced Quadricorrelator**

The output of the Balanced Quadricorrelator (see figure 8) and $\Delta w$ have a direct relationship. In this case, additional filters are not to be used for binary levels recovering. The detection block involves discrete-time differentiators and two multipliers. This configuration removes, in theory, any oscillating term at the output.

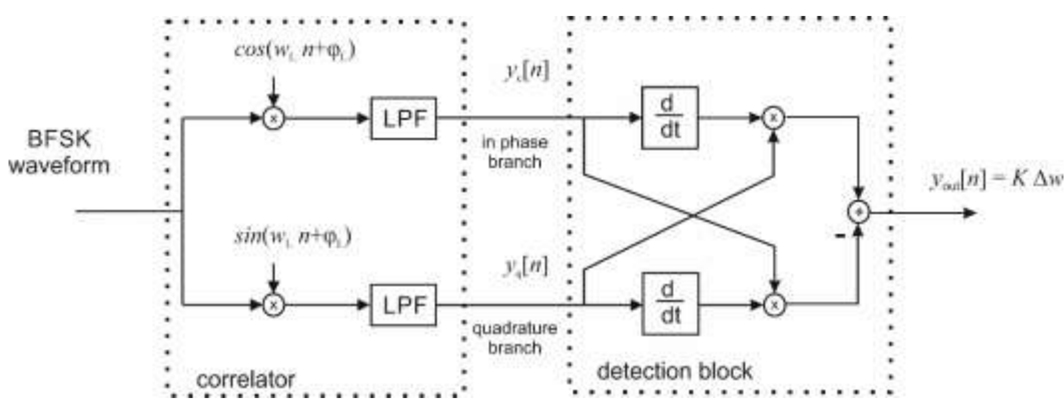


Figure 8: Balanced Quadricorrelator block scheme.

Figure 9 shows the response of the Balanced Quadricorrelator with BFSK waveform, the input of the system is depicted in a) while the output is depicted in b). The output oscillations in this case have less magnitude than those given at the Quadricorrelator, shown in 7 c).

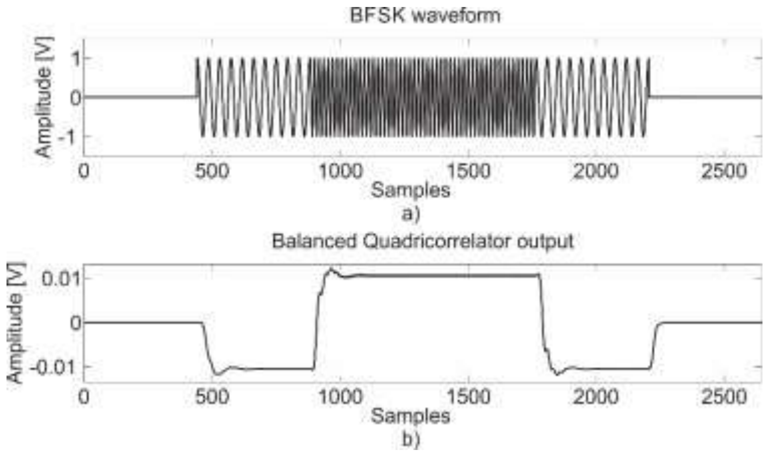


**Figure. 9. Graphical example of the Balanced Quadricorrelator performance. a) BFSK signal. b) Output of the Balanced Quadricorrelator. Low pass filter implemented through an elliptic IIR filter of 9th order, Differentiator filter implemented by a Kaiser window of fifth order. Parameters: $w_0 = 0,3562$ [rad], $w_1 = 0,1425$ [rad], $N_{samp} = 441$ samples, sampling frequency $f_s = 44100$ Hz.**

**Quotient Detector**

The block scheme of the Quotient Detector is depicted in figure 10. In this case, the detection block is the same for the Quadricorrelator except by the divisor operation included in the Quotient Detector. The analytic description of the Quotient Detector output does not reveal any oscillating term, however the division procedure produce to many glitches at the system output.

Figure 11 shows the system response to a BFSK waveform. In average, the low and high levels at the output, as shown in 11 b), are obtained. However the peaks give difficulties to distinguish one from another. However, this problem can be solved by using an additional filter at the output, having the same cutoff frequency as for the Quadricorrelator.

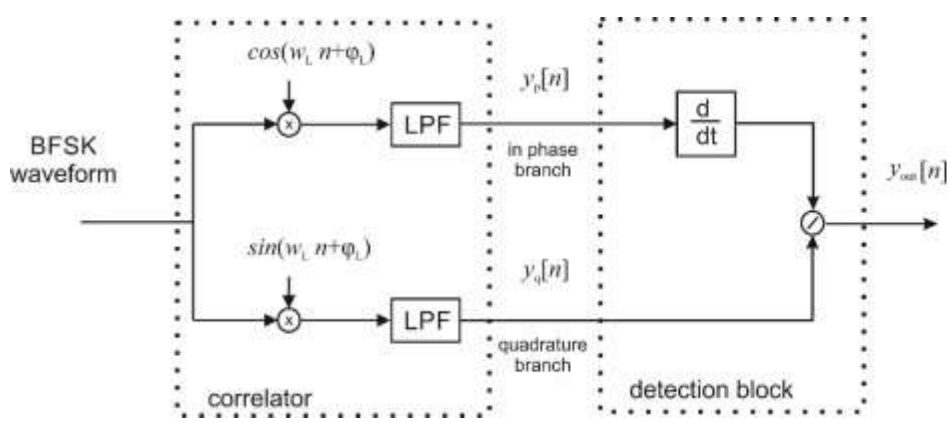

**Figure 10: Quotient Detector block scheme.**

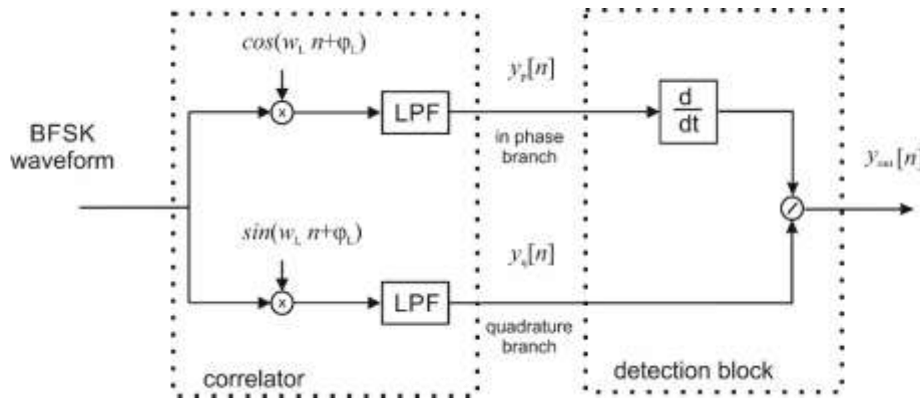


**Figure 11: Graphical example of the Quotient Detector performance. a) BFSK signal. b) Output of the Balanced Quadricorrelator. Low pass filter implemented through an elliptic IIR filter of 9th order, Differentiator filter implemented by a Kaiser window of fifth order. Parameters: $w_0 = 0,3562$ [rad], $w_1 = 0,1425$ [rad], $N_{samp} = 441$ samples, sampling frequency $f_s = 44100$ Hz.**

## CONCLUSIONES.

In regard to the correlation schemes presented, the Discrete and the Analog Correlators exhibits the highest SNR, and, as a matter of fact, the former is the less complex. Moreover, the Digital Correlator is the only that can be implemented through digital techniques, and, at the same time, under an SNR comparable to that of the Direct Correlator. For this reason, the interest on this work is to implement a Digital Correlator. Digital techniques provides flexibility, stability and reliable reproduction, whereas the Discrete Correlator can be implemented by using just one accumulator and achieves the best SNR parameter.

All BFSK correlaters-based asynchronous receivers are implemented in order to extract the instantaneous frequency of the received signal. For this reason, the use of filters and differentiators is almost an obligation. Taking into account the total number of elements used for implementing these receivers, the Balanced Quadricorrelator is the scheme with less hardware complexity. The Quadricorrelator and the Quotient Detector comprises three lowpass filters and one differentiator, whereas the Balanced Quadricorrelator involves the use of two filters and two differentiators. In addition, low pass filters are obtained at an order higher than order for differentiator filters; indeed, differentiator operation can be implemented with a fifth order filters.

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